RESEARCH SCRIPT

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ON HAMILTONIAN CYCLE OF PLANAR 3 TREES

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Abstract— *A k-tree is a chordal graph all of whose maximal cliques are the same size k + 1 and all of whose minimal clique separators are also all the same size k. If a 3-Tree chordal graph G has a planar embedding, then it is called as planar 3-Trees. A planar graph is a graph that can be embedded in the plane. Given a chordal 3-tree G , compute an embedding on the plane without edge crossings. In this work, we investigate local properties that provide information about the global cycle structure of a graph. There exists a closed walk on the graph along the edges such that visited each vertex exactly once and cover all the vertices in a single closed walk. We present a linear time algorithm to characterize the Hamiltonianicity of a 3-tree, and polynomial time algorithm to recognize the Hamiltonian circuit in the planar 3-tree. We begin by defining the global cycle properties that we shall consider. The order (number of vertices) of a graph G is denoted by n. A graph G is Hamiltonian if G has a cycle of length n. We emulate flexibility and feasibility by giving more features into the UI. Primary objective is to establish a structural properties and characterization of planar 3-trees and Hamiltonian cycle. Hamiltonian cycle has more characterization on planar 3-trees with respect to simplicial ordering and perfect elimination ordering.*

Keywords—Graph, Embedding, Hamiltonian Cycle, Planar 3-Tree

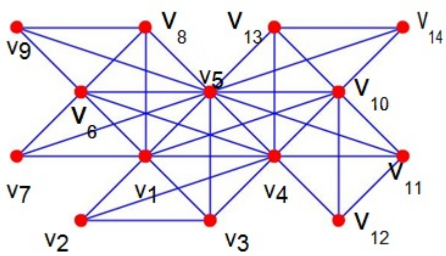
1. INTRODUCTION

A. Chordal Graph:

A clique is a maximal set of pair wise adjacent vertices. The family of cliques of G is denoted by C(G). For v ∈ V, C_v will denote the family of cliques containing v. All graphs considered will be assumed to be connected. A set S ⊆ V is a uv-separator if vertices u and v are in different connected components of G-S. It is minimal if no proper subset of S has the same property. The vertex subset S is a minimal vertex separator if there exist two non-adjacent vertices u and v such that S is a minimal uv-separator. The vertex subset S will denote the family of all minimal vertex separators of G. Chordal graphs were defined as those graphs for which every cycle of length greater than or equal to four has a chord.

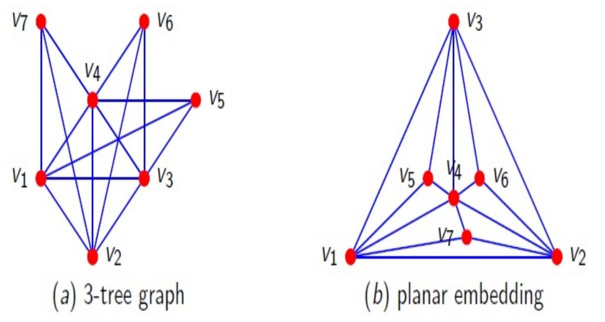
B. Planar 3-Tree:

A k-tree is a chordal graph all of whose maximal cliques are the same size k + 1 and all of whose minimal clique separators are also all the same size k. A k-path is a k-tree with maximum degree 2k, where for each integer j,k: j < 2k, there exists a unique pair of vertices, u and v, such that deg(u) = deg(v) = j. A clique tree is more structured than the chordal graphs due to its clique width property.



C. 3-Trees.

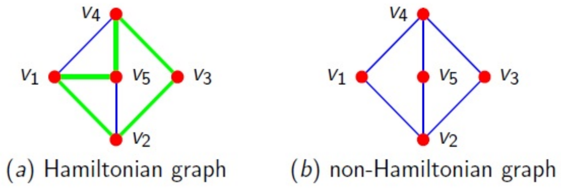
If a 3-Tree chordal graph G has a planar embedding, then it is called as planar 3-Trees. A planar graph is a graph that can be embedded in the plane. Given a chordal 3-tree G , compute an embedding on the plane without edge crossings. There are finitely many embedding for every finite chordal 3-trees. A planar 3-tree is a maximal planar graph has an embedding without edge crossings.



Planar 3-tree and its embedding.

D. Hamiltonian Cycle:

The development of graph theory has been profoundly influenced by the evolution of the internet and resulting large communication networks. Of particular interest are global properties of such networks that can be deduced from their local properties. In this work we investigate local properties that provide information about the global cycle structure of a graph. A Hamiltonian path (or traceable path) is a path in an undirected or directed graph that visits each vertex exactly once. A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian path that is a cycle. There exists a closed walk on the graph along the edges such that visited each vertex exactly once and cover all the vertices in a single closed walk.



Hamiltonian and non-Hamiltonian graphs.

E. Simplicial Ordering:

Let Δ denote the simplex category. The objects of Δ are nonempty linearly ordered sets of the form [n] = {0, 1, ..., n} with n≥0. The morphisms in Δ are (non-strictly) order-preserving functions between these sets. A simplicial set X is a contra variant function X: Δ → Set where Set is the category of small sets. (Alternatively and equivalently, one may define simplicial sets as covariant functors from the opposite category Δop to Set.) Simplicial sets are therefore nothing but presheaves on Δ. Alternatively, one can think of a simplicial set as a simplicial object (see below) in the category Set, but this is only different language for the definition just given. If we use a covariant functior X: Δ → Set instead of a contra variant one, we arrive at the definition of a cosimplicial set. Simplicial sets form a category, usually denoted s Set, whose objects are simplicial sets and whose morphisms are natural transformations between them.

2. ALGORITHMS

Simplicial Ordering:
Input : G=(V,E)
Output : A function f:V □N
Integer count =0;
While (not every vertex visited) do
Find a vertex u ϵ v such that
d(u)=3 and G[u] is k4.
f(u) =count;
count++;
mark v as visited
end while;
output f;

Simplicial Leveling:
Input: G=(V,E)
Output: A function l:N □N
Integer count =1;
H=G;
while H is not null graph do
list ϕ
for each vϵv[H] do
if deg(u)==3 then
list = list µ {u};
end if;
end for;
for vertex u ϵ list do
l(u) = level;
end for;
level++;
H=H-v{list];
end while;
output l;

Hamiltonian Cycle:
Input: G=(V,E).
Simplicial ordering f:v □N
Simplicial leveling l:v □N
Output: A cycle C of ordering n vertices
Cycle c ϕ
Choose a vertex uϵV such that
L(v) is maximum and
F(v) is minimum
Mark v as visited
While not every vertex visited do
Let v be the current vertex
Choose a vertex U ϵN(u) such that
L(u)<= l(v) and f(u)<= f(u) or
L(u)<=l(v) and f(u)>=f(v)
Mark u as visited
Push u on top of v on c
Continue while loop as u as current vertex
And while
Output C.

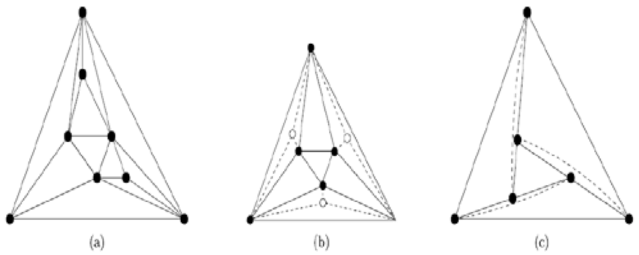
A. Theorem 1.

Existence of a Hamiltonian path H(u,v) in _G with δ(u, v) _ 3 implies that _G is Hamiltonian.
Existence of a Hamiltonian path in a graph _G is ensure.
Lemma 3.2. For H(u,v) in _G, δ(u, v) _ 3.
Proof.
Let us assume for clarity of arguments that u is denoted by 1 and v is denoted by n and all vertices along Hamiltonian path H(u,v) in _G are denoted by 2, 3, . . . , n − 1 and it will be used throughout this paper. We interchangeably use u, v with 1,n respectively. We prove Lemma 3.2 by contradiction. Let us assume that δ(u, v) _ 4.

B. Theorem 2:

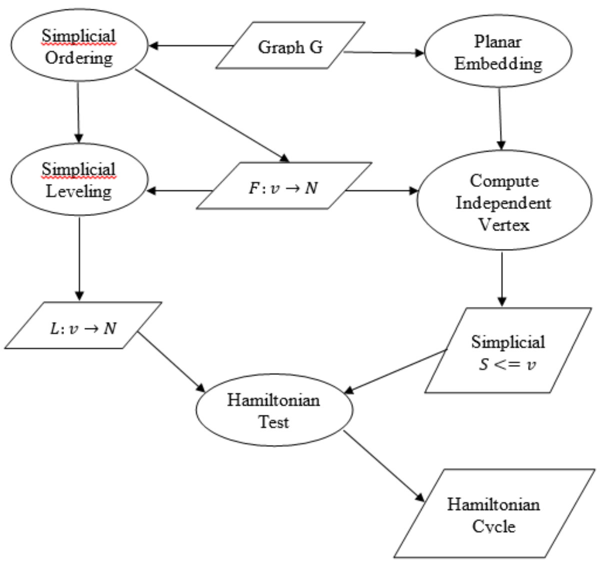
Let G = (V ,E) be a connected graph with n vertices such that for all pairs of distinct nonadjacent vertices u, v ∈ V we have d(u) + d(v) + δ(u, v) _ n+1. Then G has a Hamiltonian path .It will be shown in this paper that famous Ore's conditions. Also the introduction of the parameter δ(u, v) it seems to be significant with respect to the related degree conditions for Hamiltonian paths and cycles in graphs. The rest of the paper is organized as follows. We conclude by introducing some open problems for future research.

Beyond Partial 3-Tress:
In this section, we give some examples of graphs where no realization with rational coordinates is possible, hence providing counter-example to some possible conjectured generalizations. The first example is the octahedron where all interior face areas are 1 except for two non-adjacent, non-opposite faces, which have area 3. Any drawing that respects these areas must have some complex coordinates.



See Fig. (a) for an illustration of this graph. Note that both the octahedron and G1 are planar partial 4-trees, so not all partial 4-trees have equiareal drawings. The second example is the octahedron where all interior face areas are 1 except that the three faces adjacent to the outer-face have area 3. (Alternatively, one could ask for an equiareal drawing of graph G2 in Fig. (b).) Assume, after possible linear transformation, that the vertices in the outer-face are at (0, 0), (0, 13) and (2, 0). Thus even if a partial 4-tree has an equiareal drawing, it may not have one with rational coordinates.The third example is again the octahedron, with three of the interior face areas prescribed to be 0, which forces some edges to be aligned as shown in Fig. (c). If all other interior faces have area 1/8, and the outer-face is at (1, 0), (0, 1),
(0, 0), then similar computations show that some of the coordinates of the other three vertices are $(3\pm\sqrt{5})/8$. If the edges that appear dotted in the figure are removed, we obtain a graph that is a crucial ingredient in.

3. Architecture Diagram for Proposed System



For finding Hamiltonian cycle on planar 3-tree graph two different methods to be followed they are simplicial ordering and level elimination, in simplical ordering process we have to identify the number of 3-cliques in the planar 3-tree graph,3-clique of an graph can to identify by the vertex whose having degree less than 3 that vertex is known as simplical vertex ,in simplicial ordering we have name the simplicial vertex present in the graph, using level Elimination process remove the vertices are in simplicial order, the process to be continue until the 3-tree graph gets no simplicial vertex.

4. RESULTS AND DISCUSSION

Let V^' be the vertex in N_2 (u)\N_1 (v). We distinguish the following cases. In case 1, u has no neighbor in S_1 (G)\\{v}.By the induction hypothesis, there is a hamiltonian cycle C in G –v. By (1), there exists atleast one edge ux ∈ E(C)∩E(G[N_1 (v)]). Now replacing ux in C by the path uvx,the resulting cycle is a Hamiltonian cycle of G. Case 2, u has neighbor in S_1 (G)\{v}. By Lemma 9,N_1 (ω)⊆N_2 (u)∪{u} for every ω ∈(S_1 (G){v})∩ N_1 (u). If u has atleast two neighbors in S_1 (G)\{v}, then when we delete all k+1 vertices of N_2 (u)∪{u},we will obtain 4 components except for the unique case that n=k+4=7. In the former case we obtain a contradiction, since τ(G)≥ (k+1)/3. Hence u has exactly one neighbor in S_1 (G)\{v}. In the latter exceptional case, and one can easily find a hamiltonian cycle of G. Hence we now suppose n>=k+5, and we let N_1 (u)\N_2 (u)={v,w}. Using that G is a (k+1)/3 -tough graph, by Lemma 9,v'w∈E(G);otherwise N_1 (w)=N_1 (v), and if we delete all k vertices of N_1 (w), we obtain atleast three components, contradicting that G is (k+1)/3 - tough.
By the induction hypothesis, G-{v,w} has a Hamiltonian cycle C, implying that u has two neighbors x,y in C. If v' ∈{x,y}, then v" ∈({x,y}\{v^'}) is a vertex contained in C with v"v ∈E(G), and we replace the path v' uv" by v'wuvv"; v^'∉{x,y}, then there exists at most one vertex in {x,y}\N_1 (w), say y∈N_1 (w), and we replace the path xuy by xvuwy. In both cases the resulting cycle is a Hamiltonian cycle of G. In Theorem [11], if G is a graph on n≥3 vertices with δ(G)>n/2, then G is Hamiltonian We now have all the ingredients to prove the following generalization of the consequence of theorem 7 for 2-tree. In Theorem [12], if G≠K_(2@) is a k+1/3 w tough K-tree(K≥2), then G is Hamiltonian. Proof by Theorem 7 or its Consequence for 2-trees, we only need to consider the case that k>=3. We proceed by induction on n=|V(G)| Obviously δ(G)= k. Hence using theorem 11,we obtain that if either 4≤k≤n≤k+4 or 3=k≤n≤k+3=6 , then G is Hamiltonian
Suppose next that either n≥n+5 or n=k+4=7, and that H is Hamiltonian for any k+1/3 tough k-tree H with fewer than n vertices By lemma 10, it suffices to consider the case thatS_2 (G)≠∅ .for any u∈ S_2 (G) , by lemma 8, there exists a vertex v∈ S_1 (G) such that uv ∈ E(G) Since u∈ S_2 (G) and the N_1 (V)clique contain u |N_2 (u) \ N_1 (v)|=k-1,Hence |N_2 (u) \ N_1 (v)|=1. In Lemma 6,Let G≠K k be a K-tree(k≥2).Then

G is k-connected;

S_1 (G)≠Ø;

S_2 (G) is an independent set;

τ(G-v)≥τ(G) for a k-simplicial vertex vϵS_1 (G);

Every k-simplicial vertex(if any)of G-S_1 (G) is adjacent in G to atleast one vertex of S_1 (G).

τ(G-S_1(G)≥τ(G).

5. PROOF

This follows immediately from the definition;
This follows immediately from the definition;

If not, then for some adjacent vertices u,v ∈S_1 (G),u is a k-simplicial vertex of G-v with degree d(u)<k, a contradiction;

If u is a k-simplicial vertex of G-S_1 (G),i.e. with d_(G-s_1(G)) (u)=k,then d(u)>k;since u∉S_1 (G).Hence the claim follows,

Suppose, to the contrary, that S is a tough set of G-v such that τ(G-v)=|s|/(ω((G-v)-S)<τ(G).Then v is adjacent to vertices in atleast two components of (G-v)-S, contradicting the fact that all neighbors of v are mutually adjacent (in G and hence in G-v).This completes the proof, this is a consequences of(v).

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