

# Selective Harmonic Elimination for Single Phase Inverter using Pattern Search Method

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**Abstract** — As saving of electrical energy is the main target of industrial consumer and also utility, the use of energy efficient equipment increases rapidly. The modern industrial system utilize variable speed drives through thyristor converters as the control becomes simple, more efficient, accurate and widely available. However, it gives rise to a varied harmonic spectrum on the ac power systems and pollutes the system. Therefore some technique is needed to eliminate the dominant harmonics present in the system. Hence Selective Harmonic Elimination (SHE) or programmed harmonic elimination for an single phase inverter fed induction motor drive is considered in this paper. The main objective of selective harmonic elimination is to produce the fundamental while not generating specifically chosen harmonics. The output voltage waveform of an inverter is given by Fourier series expansion which contains both the fundamental and the harmonics. Suitable technique has to be used to solve the transcendental equation. Traditional optimization methods suffer from various drawbacks, such as prolonged and tedious computational steps and convergence to local optima, thus, the more the number of harmonics to be eliminated, the larger the computational complexity and time. So in this research work, Pattern Search (PS) method has been employed to eliminate the harmonics. The complete solutions for 3-level unipolar switching patterns to eliminate the 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics are given.

**Keywords** — Selective Harmonic Elimination, Unipolar Pulse Width Modulation, Pattern Search Method, Total Harmonic Distortion.

## 1. INTRODUCTION

Selective Harmonic Elimination (SHE) control has been a widely researched alternative to traditional PWM techniques. The typical goal of SHE-PWM is to generate a train of pulses such that fundamental component of the resultant waveform has a specified frequency and amplitude [19]. This technique consists of synthesizing a PWM waveform by setting its pulse pattern properly to eliminate selected order of harmonics. Hence, roughly speaking, the objective of SHE-PWM is to push most harmonic energy into high-frequency regions such that low-frequency harmonics are well attenuated [20]. This is accomplished by solving transcendental equations which are non-linear in nature [8]. This technique theoretically offers the highest quality of output waveform.

A sinusoidal PWM inverter, which is a DC-AC power inverter, is used for a wide variety of applications because of its flexibility in driving frequency and voltage in the power electronics field. If the switching frequency is not high, but the control accuracy is good, off-line PWM control is efficient because it optimizes PWM waveforms for harmonic elimination and total harmonic distortion.

Two major advantages of applying SHE technique are:

1) If the inverter is used to supply AC power of constant frequency to general AC loads, a filter is usually installed at its output. In this case, when low-order harmonics are eliminated through the modulation of the inverter, only high-order harmonics will be present at the output and need to be attenuated by the filter. The cut-off frequency of the filter can thus be increased, leading to a significant reduction of the filter size and cost.

2) When used in an AC drive system, eliminating the low-order harmonic voltages leads to great reduction of low-order harmonic torques of the motor [19].

The transcendental equations are solved using techniques such as iterative numerical techniques [1], Selective harmonic elimination [6], Programmed PWM technique [7], elimination using resultants [16], etc., to compute the switching angles. However, the drawback of these methods is heavy computation burden and complicated hardware [8]. The main challenge of solving the associated nonlinear equation, which are transcendental in nature and therefore have multiple solutions, is the convergence. It is generally accepted that the performance of an inverter, with any switching strategy, can be related to the harmonic contents of its output voltage [17]. Here a method is presented to solve the transcendental equations whose solutions are highly reliable with fast converging characteristics.

Direct Search (DS) methods are evolutionary algorithms used to solve constrained optimization problems. DS method does not require any information about the gradient of the objective function at hand, while searching for an optimum solution. This family includes PS algorithms, Simplex Methods (SM) (different from the simplex method used in linear programming), Powell Optimization (PO) and others. The PS method is a technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Generally, PS has the advantage of being very simple in concept, easy to implement and computationally efficient. Hence it can be used to solve the transcendental equations involving harmonics.

2. PROBLEM FORMULATION

In this section, the mathematical formulation of the problem is presented for unipolar case.

2.1. Unipolar PWM scheme

The Fourier series expansion of unipolar waveform [13] is given by :

$$v(\omega t) = (4V_{dc} / \pi) \left\{ \sum_{n=1,3,5}^{\infty} \sin(n\omega t) / n \right. \\ \left. [(\cos n\theta_1 - \cos n\theta_2 + \cos n\theta_3 - \cos n\theta_4)] \right\} \quad (1)$$

The problem is to find the unknown angles with transcendental equations as follows:

$$\begin{aligned} \cos\theta_1 - \cos\theta_2 + \cos\theta_3 - \cos\theta_4 &= m & (2) \\ \cos 5\theta_1 - \cos 5\theta_2 + \cos 5\theta_3 - \cos 5\theta_4 &= 0 & (3) \\ \cos 7\theta_1 - \cos 7\theta_2 + \cos 7\theta_3 - \cos 7\theta_4 &= 0 & (4) \\ \cos 11\theta_1 - \cos 11\theta_2 + \cos 11\theta_3 - \cos 11\theta_4 &= 0 & (5) \end{aligned}$$

This formulated problem will be solved using PS method whose objective function aims to minimize the harmonic equations subject to the constraints [11-13],

$$0^\circ < \theta_1 < \theta_2 < \theta_3 < \theta_4 < 90^\circ \quad (6)$$

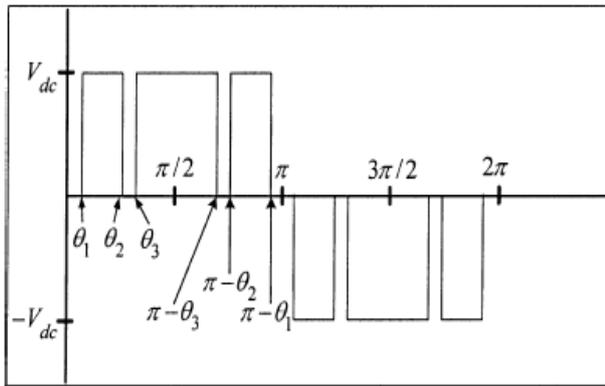


Fig. 1. Unipolar PWM waveform

3. PATTERN SEARCH METHOD

The PS optimization routine is a derivative free evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Generally, PS has the advantage of being very simple in concept, and easy to implement and computationally efficient algorithm [11]. Unlike other heuristic algorithms, such as GA, PS possesses a flexible and well-balanced operator to enhance and adapt the global and fine tune local search.

The PS algorithm proceeds by computing a sequence of points that may or may not approach the optimal point. The algorithm starts by establishing a set of points called *mesh*, around the given point. This current point could be the initial starting point supplied by the user

or it could be computed from the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a *pattern*. If a point in the mesh is found to improve the objective function at the current point, the new point becomes the current point at the next iteration.

This may be better explained by the following:

**First Step:** The Pattern search begins at the initial point  $X_0$  that is given as a starting point by the user. At the first iteration, with a scalar of magnitude 1 called *mesh size*, the pattern vectors are constructed as  $[0 \ 1]$ ,  $[1 \ 0]$ ,  $[-1 \ 0]$ , and  $[0 \ -1]$ , they may be called direction vectors. Then the PS algorithm adds the direction vectors to the initial point  $X_0$  to compute the following mesh points:

$$\begin{aligned} X_0 + [1 \ 0] \\ X_0 + [0 \ 1] \\ X_0 + [-1 \ 0] \\ X_0 + [0 \ -1] \end{aligned}$$

Fig. 3 illustrates the formation of the mesh and pattern vectors. The algorithm computes the objective function at the mesh points in the order shown.

The algorithm polls the mesh points by computing their objective function values until it finds one whose value is smaller than the objective function value of  $X_0$ . If there is such point, then the poll is successful and the algorithm sets this point equal to  $X_1$ .

After a successful poll, the algorithm steps to iteration 2 and multiplies the current mesh size by 2, (this is called the *expansion factor* and has a default value of 2). The mesh at iteration 2 contains the following points:  $X_1 + 2*[1 \ 0]$ ,  $X_1 + 2*[0 \ 1]$ ,  $X_1 + 2*[-1 \ 0]$  and  $X_1 + 2*[0 \ -1]$ . The algorithm polls the mesh points until it finds one whose value is smaller than the objective function value of  $X_1$ . The first such point it finds is called  $X_2$ , and the poll is successful. As the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration.

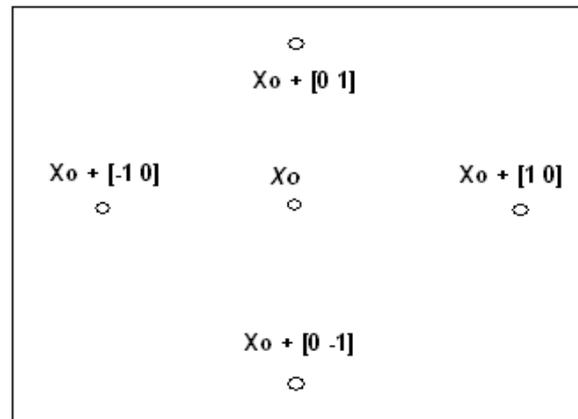


Fig. 2. PS Mesh points and Pattern

**Second Step:** Now if in iteration 3, none of the mesh points has a smaller objective function value than the value at  $X_2$ , the poll is called an unsuccessful poll. In this case, the algorithm does not change the current point at the next iteration, i.e.,  $X_3 = X_2$ . At the next iteration, the algorithm

multiplies the current mesh size by 0.5, a contraction factor, so that the mesh size at the next iteration is smaller. The algorithm then polls with a smaller mesh size. The PS optimization algorithm will repeat the illustrated steps until it finds the optimal solution for the minimization of the objective function. The PS algorithm stops when any of the following conditions occurs:

- The mesh size is less than the mesh tolerance.
- The number of iterations performed by the algorithm reaches the value of maximum iteration number.
- The total number of objective function evaluations performed by the algorithm reaches the value of maximum function evaluations.
- The distance between the point found at one successful poll and the point found at the next successful poll is less than the specified tolerance.
- The change in the objective function from one successful poll to the next successful poll is less than the objective function tolerance.

All the parameters involved in the PS optimization algorithm can be pre-defined subject to the nature of the problem being solved.

4. OPTIMIZATION RESULTS

To obtain the best and optimum solution for the given non-linear transcendental equations, the following PS parameters were used.

Mesh tolerance: 1e-006

Maximum iterations: 100 \* no. of variables

Maximum function evaluations: 2000 \* no. of variables.

Using the proposed PS technique the equations of unipolar case is solved and the four switching angles were found for  $m = 0.8$ . The plots obtained using pattern search tool is given below.

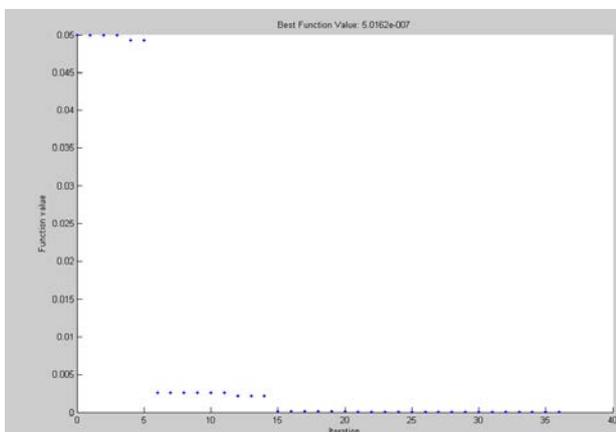


Fig. 3. Best function value Vs iteration for m= 0.8

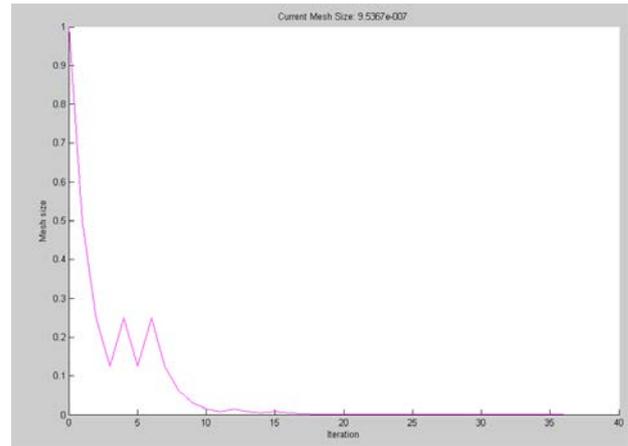


Fig. 4. Current mesh size Vs iteration for  $m = 0.8$   
This process can be repeated for various modulation indices from 0.1 to 1.3 for both unipolar and bipolar cases.

5. SIMULATION RESULTS

Let us consider the following example, inverter circuit to produce unipolar output pattern. It is connected with an split phase induction motor load as shown below. Simulations were carried out on a Intel Core 2 Duo 1.8GHz, 512 MB RAM processor. The rating of the proposed AC drive system was

- DC power supply : 325 V
- Load : Split phase induction motor
- Hp : 0.25
- Voltage (rms) : 230 V
- Frequency : 50 Hz
- Modulation index : 0.8

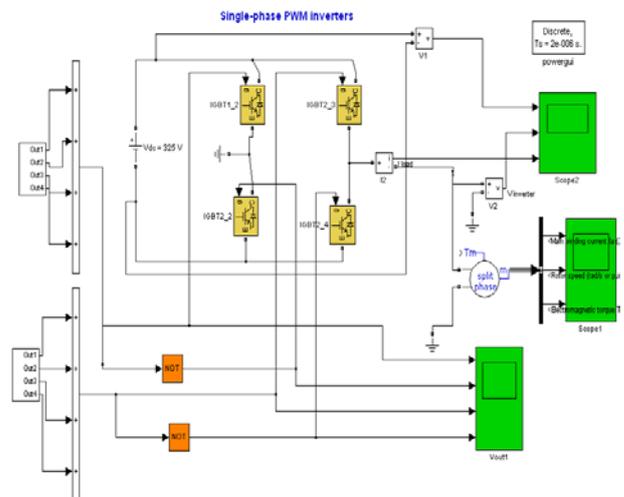


Fig . 5. Simulation Diagram for elimination of selected harmonics

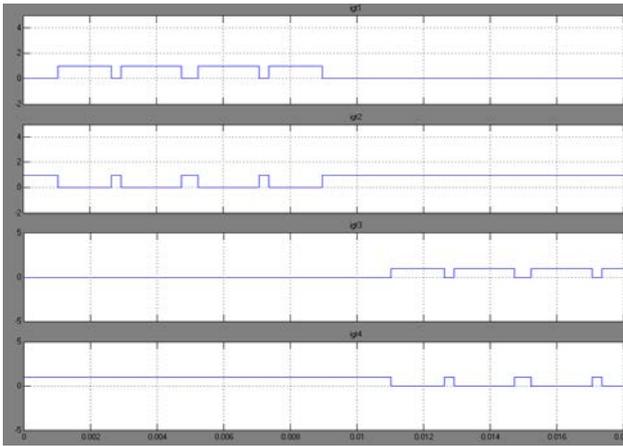


Fig. 6. Unipolar switching pulse pattern

The switching pulse pattern required to eliminate the 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics in an single phase inverter's output waveform is given in Fig. 6.

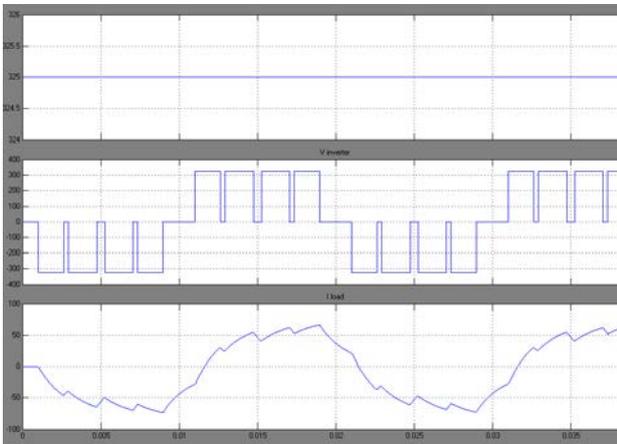


Fig. 7. Unipolar output voltage and current waveforms

Fig.7. shows the output voltage and current waveforms obtained after the application of switching angles calculated using pattern search method.

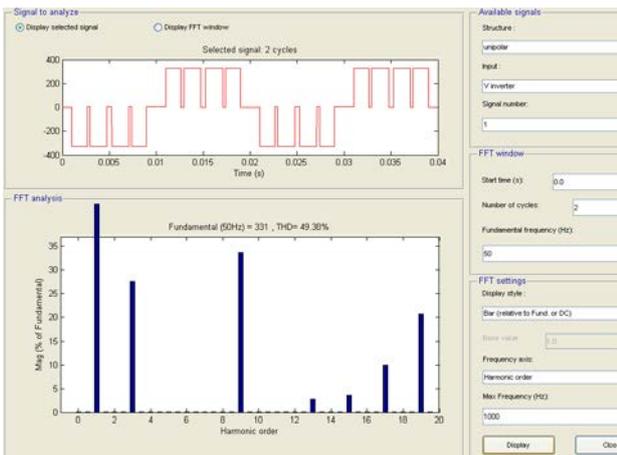


Fig. 8. FFT after implementation of obtained solution

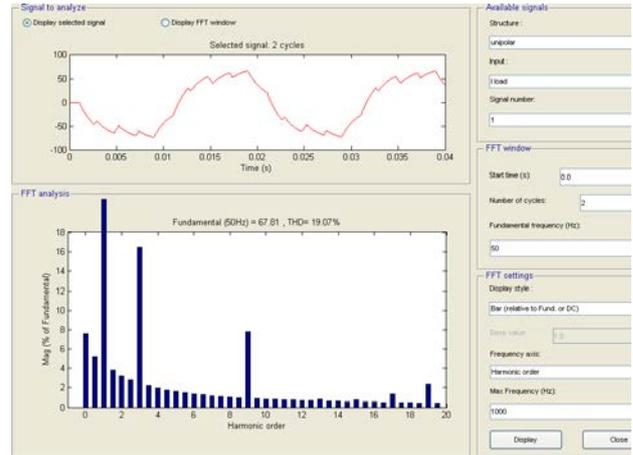


Fig. 9. Load current waveform with its harmonic pattern

Fig.8. shows the various harmonic levels present in the output voltage waveform. Elimination of 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics reduces THD to 49.38%. Further the THD value can be reduced by eliminating more number of harmonic orders. This will in turn increases the number of estimated switching angles. Fig. 9. shows the presence of various harmonic levels in the load current waveform. The percentage THD of this waveform is 19.07%. By designing suitable filter further reduction in %THD is possible.

## 6. CONCLUSION

Pattern Search method of determining the switching angles of the programmed PWM single phase inverter circuit is presented. PS method is used to find the optimum switching angles based on minimization of objective function. The estimation up to 4 switching angles per quarter cycle is performed while minimizing a pre-selected number of harmonics. The validity of the estimated angles has been verified from the THD and the FFT of the resultant waveforms. Further work should focus on real time implementation of SHE-PWM inverters.

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